Patient Admission and Bed Allocation Policies in Acute Care Wards:

An Application to a Neurology Ward

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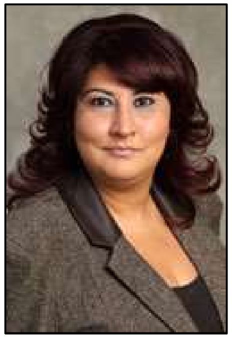

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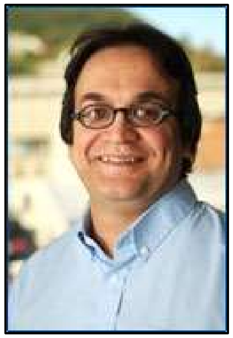

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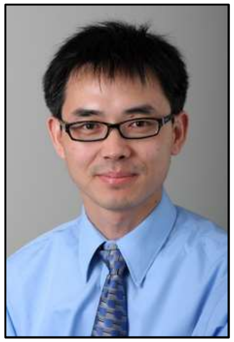


Outline

- Background
- Problem description
- Dynamic program formulation
- Solution methodology
  - Static bed allocation model
  - Approximate dynamic program
- Heuristic admission polices
- Policy performance evaluation
- Summary/conclusion

Background

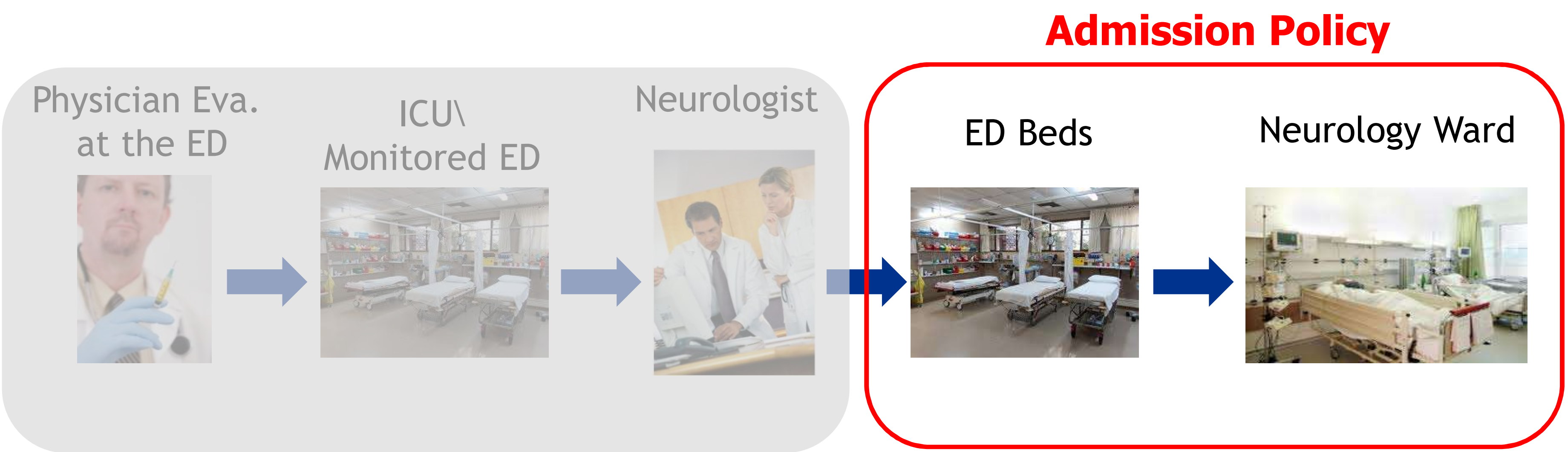
Neurology Ward

- Neurology ward at Montreal Neurological Hospital (MNH)
- Specialized (and time-sensitive) treatment in the neurology ward
- Delay in access to care has negative impact on the patient's health and treatment outcome.[1]
- Transfer patient to another hospital
- Off-unit service is not available.
- Multiple types of patients with different medical characteristics (stroke & non-stroke, mild & severe):
  - Impact of delay in receiving care on deterioration of health.
  - Arrival rates and length of stay in the ward.

[1]Kucukyazici et al. (2010)

Patients Flow

- Admission through ED accompanied by extensive medical examinations, followed by decision for admission to neurology ward.



Admission Policy

- *Rules* for allocation of inpatient beds among multiple types of patients as well as patient transfers.
- Trade-off 1: (prioritization)

Deterioration in Functionality	vs.	Blocking more Admissions

- Trade-off 2: (transfer)

ED Boarding Time	vs.	Inconveniences of transfer

- A Dynamic Programming model to find the best admission policy.

Problem Description

Problem Description

- Patient type: $i \in \{1, \ldots, n\}$
- Patients wait in the ED to be admitted to the ward.
- Waiting in the ED incurs some cost in terms of patient's health status.
- Opportunity cost of waiting per unit time for type-i patients is π_i.
- Total number of beds in ward is B.
- Each bed can be used for any patient type.

Problem Description

- Type-i patient's arrival:
  - Poisson process with rate of λ_i per unit time
- Decision for a new patient: Let the patient enter the system or transfer to another hospital.
- Transfer cost for type-i patients: κ_i
- Decision for a waiting patient: Admit to a bed or keep the patient waiting.
- Type-i patient's length of stay (LOS):
  - Exponential with mean of $\frac{1}{\mu_i}$.

The Model

- Continuous time model over infinite horizon.
- Decisions are only made when there is a change in the system state.
- This change can be arrival or discharge of a patient.
- Minimize **the average total cost** of waiting and transferring in each period.

The Dynamic Program

System State

- Two variables for each type of patients:
  - Number of type-i patients waiting: x_i
  - Number of occupied beds by type-i patients: b_i
- Define $\boldsymbol{x} = (x_1, \dots, x_n)^{\mathrm{T}}$ and $\boldsymbol{b} = (b_1, \dots, b_n)^{\mathrm{T}}$
- State of system: $(\boldsymbol{x}, \boldsymbol{b})$
- ED capacity: K
- Finite state space: $\sum_{i=1}^{n} x_i \leq K, \sum_{i=1}^{n} b_i \leq B.$

Decisions

- In case of an arrival:
  - Transfer the patient to another hospital.
  - Let the patient join the queue (and do nothing else).
  - Admit the patient to the ward.
- In case of a discharge:
  - No admission (leave the system as is).
  - Admit one patient from queue to the ward (if there is any waiting patient).

The Optimality Equation

ρ^* = Optimal average cost per period.

$h(\boldsymbol{x},\boldsymbol{b})$= Relative value function or bias function if we start from state $(\boldsymbol{x},\boldsymbol{b})$.

$$h(\boldsymbol{x},\boldsymbol{b}) = \boldsymbol{\pi}^{\mathrm{T}}\boldsymbol{x} - \rho^* + \sum_{i=1}^{n} \lambda_i \min_{(a_i,t_i)\in\mathfrak{A}_i(\boldsymbol{x},\boldsymbol{b})} [t_i\kappa_i + h(\boldsymbol{x} + (1-a_i-t_i)\boldsymbol{e}_i, \boldsymbol{b} + a_i\boldsymbol{e}_i)]$$
$$+ \sum_{i=1}^{n} b_i\mu_i \min_{\boldsymbol{d_i}\in\mathfrak{D}_i(\boldsymbol{x},\boldsymbol{b})} [h(\boldsymbol{x}-\boldsymbol{d_i}, \boldsymbol{b}-\boldsymbol{e}_i+\boldsymbol{d_i})]$$
$$+ \left(1 - \sum_{i=1}^{n}(\lambda_i + b_i\mu_i)\right) h(\boldsymbol{x},\boldsymbol{b}); \quad \forall(\boldsymbol{x},\boldsymbol{b}).$$

Data & Model Parameters

Arrival and LOS

- Three full years of data from MNH (Patient registry system, patients' chart review, ED information system).
- Arrivals: χ^2 goodness-of-fit test.
- LOS: Anderson-Darling goodness-of-fit test.

Patient Type	Arrival Process (per day)			Length of Stay (day)		
	Mean (λ)	Variance	H_0 : Arrival is Poisson	Mean ($^1/_\mu$)	Variance	H_0 : LOS is Exponential
Mild Non-Stroke	0.236	0.246	Not Rejected ($0.05<p<0.10$)	13.003	162.383	Not Rejected ($0.05<p<0.10$)
Mild Stroke	0.262	0.291	Not Rejected ($0.10<p<0.25$)	11.491	114.204	Not Rejected ($0.05<p<0.10$)
Severe Non-Stroke	0.139	0.141	Not Rejected ($0.25<p<0.50$)	19.011	305.904	Not Rejected ($0.25<p<0.50$)
Severe Stroke	0.113	0.117	Not Rejected ($0.25<p<0.50$)	22.002	596.445	Not Rejected ($0.50<p$)

Waiting Cost

- Patient's health deterioration as a consequence of waiting in ED emerges as worse functionality of the patient.
- Increase in the ED boarding time is associated with increase in the probability that the patient is not discharged to home.

> *A 10% increase in the ED boarding time is related to a 7.7% increase in the probability of not being able to go home upon discharge.*[1]

- Alternative discharge destination is rehabilitation center (Rehab) or long term care facility (LTC).
- Health related Quality of Life (QoL) is lower for Rehab or LTC compared to home.[2]
- Waiting cost is defined as expected QoL lost due to increase in the ED boarding time.

[1] Kucukyazici et al. (2010)

[2] Nichols-Larsen et al. (2005)

Waiting Cost

- β_i : Increase in the probability of not being discharged to home for type-i patients as a result of one time unit of boarding in the ED.
- β_i is estimated through a regression model for each type.
- Conditional probabilities of being sent to Rehab or LTC; given the patient is not discharged to home.

	Home	**Rehab**	**LTC**
Conditional Probability	-	s_i^{R}	s_i^{L}
Quality of Life (QoL)	$\mathrm{Q_H}$	$\mathrm{Q_R}$	$\mathrm{Q_L}$

- Waiting cost for type-i patients:

$$\pi_i = \beta_i \left(\mathrm{Q_H} - \left(s_i^{\mathrm{R}}\mathrm{Q_R} + s_i^{\mathrm{L}}\mathrm{Q_L}\right)\right)$$

Transfer Cost

- Transferring Strategies:
  - Transfer the patient if the waiting time exceeds a threshold.
  - Transfer the patient before waiting starts based on state of the system.
- Transfer threshold: d_i time units.
- Waiting cost per time unit: π_i.
- Transfer cost of type-i patients: $\boldsymbol{d_i \pi_i}$.

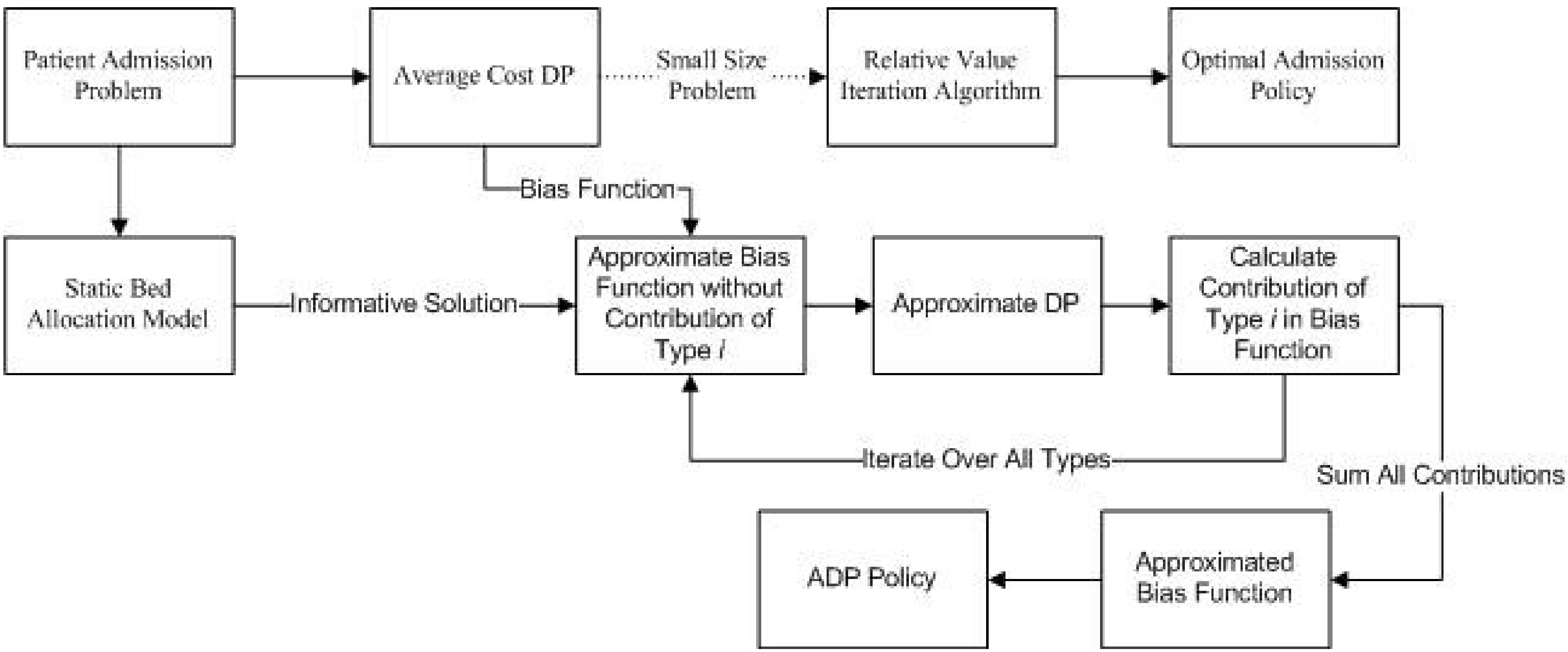
Solution Methodology

Research Question

- Standard dynamic programming techniques, such as value iteration, can be used to compute an optimal policy.
- However, such approaches only work for relative small problems due to the well-known *curse of dimensionality*.

Can we compute a good policy for large-scale problems in a reasonable amount of time?

Solution Methodology



Static Bed Allocation Model

- Allocate $\tilde{b}_i$ beds to type-i patients
- Allocate k_i of waiting room capacity to type-i patients
- Adjusted arrival rate: $\tilde{\lambda}_i (\leq \lambda_i)$
- Service rate: μ_i
- Each patient type's queue: $M/M/\tilde{b}_i/\tilde{b}_i + k_i$
- Find $\tilde{b}_i$, k_i, $\tilde{\lambda}_i$ such that the average cost of waiting and transferring is minimized

Static Bed Allocation Model

- Non-linear Program:

$$(\mathbf{SM}) \qquad \text{Min } \sum_{i=1}^{N} \pi_i {L_q}^i(\tilde{\lambda}_i, \tilde{b}_i, k_i) + \kappa_i(\lambda_i - \tilde{\lambda}_i(1 - p_{k_i}))$$

Subject to:

$$\sum_{i=1}^{n} \tilde{b}_i \leq B,$$

$$\sum_{i=1}^{n} k_i \leq K,$$

$$\tilde{\lambda}_i \leq \lambda_i,$$

$$\tilde{\lambda}_i, \tilde{b}_i, k_i \geq 0 \text{ and } \tilde{b}_i, k_i \text{ are integer}, \forall i.$$

- Non-linear mixed-integer program with linear constraints
- Relax integrality constraints appropriately

Information from SM

- Value of dual variable associated with the constraint $\sum_{i=1}^{n} \tilde{b}_i \leq B$; denoted by α.
- The value of α can be interpreted as the opportunity cost of occupying one bed per unit time.
- Average LOS of type-i patients is ${}^{1}/_{\mu_i}$.
- The opportunity cost of occupying a bed by a type-i patient is estimated as: ${}^{\boldsymbol{\alpha}}/_{\boldsymbol{\mu_i}}$

Information from SM

- Average length of queue from the solution of the SM for type-i patients is L_i^*.

- Average waiting time: $W_i^* = \frac{L_i^*}{\tilde{\lambda}_i\left(1-p_{k_i}\right)}$

- So average waiting cost of a type-i patient in the queue is estimated as: $\boldsymbol{\pi_i W_i^*}$

Approximate DP

- For a chosen type i, the bias function is approximated by:

$$\boldsymbol{h}(\boldsymbol{x},\boldsymbol{b}) \approx \boldsymbol{h_i}(\boldsymbol{x_i},\boldsymbol{b_i}) + \sum_{j \neq i} \left(\boldsymbol{\pi_j W_j^*} \left(x_j - L_j^*\right)^+ + \frac{\boldsymbol{\alpha}}{\boldsymbol{\mu_j}} \left(b_j - \tilde{b}_j^*\right)^+ \right)$$

- Plug approximate bias function into equivalent linear program.
- Relax some constraints in action sets.
- The resulting linear program is equivalent to a dynamic program with only 2 state variables.

The Resulting 2-Dimensional DP

- For type i, we solve the following DP:

$$h_i(x_i, b_i) + \rho_i^* = \pi_i x_i$$
$$+\lambda_i \min_{(a_i,b_i)\in\mathfrak{U}_i(x_i,b_i)} \{t_i\kappa_i + h_i(x_i + 1 - a_i - t_i, b_i + a_i)\}$$
$$+(1 - \lambda_i - b_i\mu_i)h_i(x_i, b_i) + \text{MIP}(x_i, b_i, h_i(x_i, b_i)),$$
$$\forall x_i \leq K, \forall b_i \leq B.$$

- Iterate over all types to find h_i ($\forall i$) functions.
- Final approximation:

$$h(\boldsymbol{x}, \boldsymbol{b}) \approx \sum_{i=1}^{n} h_i(x_i, b_i) = \tilde{h}(\boldsymbol{x}, \boldsymbol{b})$$

- Once we know the $\tilde{h}(\boldsymbol{x}, \boldsymbol{b})$, we can develop a policy (ADP policy).

The ADP Policy

- Using the estimated bias function and Original DP:

The ADP Policy:

1. In case of an arrival of type-i patient, compare the costs associated with admission of the patient to the queue (if there is space in the ED), admission to the ward (if there is available bed), and transferring to another hospital, which are $\tilde{h}(\boldsymbol{x}+\boldsymbol{e}_i, \boldsymbol{b})$, $\tilde{h}(\boldsymbol{x}, \boldsymbol{b}+\boldsymbol{e}_i)$, and $\kappa_i + \tilde{h}(\boldsymbol{x}, \boldsymbol{b})$ and choose the decision with minimum cost.

2. In the case of discharge of a type-i patient, compare the costs associated with admission of patient type j ($\forall j$) from queue (any type of which there is at least one patient waiting in the queue) and admitting no patient, which are $\tilde{h}\left(\boldsymbol{x}-\boldsymbol{e}_j, \boldsymbol{b}-\boldsymbol{e}_i+\boldsymbol{e}_j\right); \forall j: x_j \neq 0$ and $\tilde{h}(\boldsymbol{x}, \boldsymbol{b}-\boldsymbol{e}_i)$, and choose the decision with minimum cost.

Computational Experiments with Realistic Problem Instances

Two Static Policies

- Based on the solution of **SM**.

1. The **B**ed **A**llocation (BA) policy:
   - At any given time, the maximum number of occupied beds by type-i patients is $\tilde{b}_i^*$.
   - Transfer some of the new arrivals of type-i patients based on the adjusted arrival rate ($\tilde{\lambda}_i^*$).
2. The **B**id **P**rice (BP) policy:
   - Motivated by Revenue Management.
   - Compare the cost of transfer (κ_i) with the opportunity cost of occupying a bed ($\alpha\mu_i^{-1}$) if there is at least one bed available and cost of transfer to sum of opportunity cost of occupying a bed and average waiting time ($\alpha\mu_i^{-1} + \pi_i W_i^*$) is ward is full.
   - If one bed becomes available, priority is given to the patients with highest waiting cost.

A Comparative Analysis

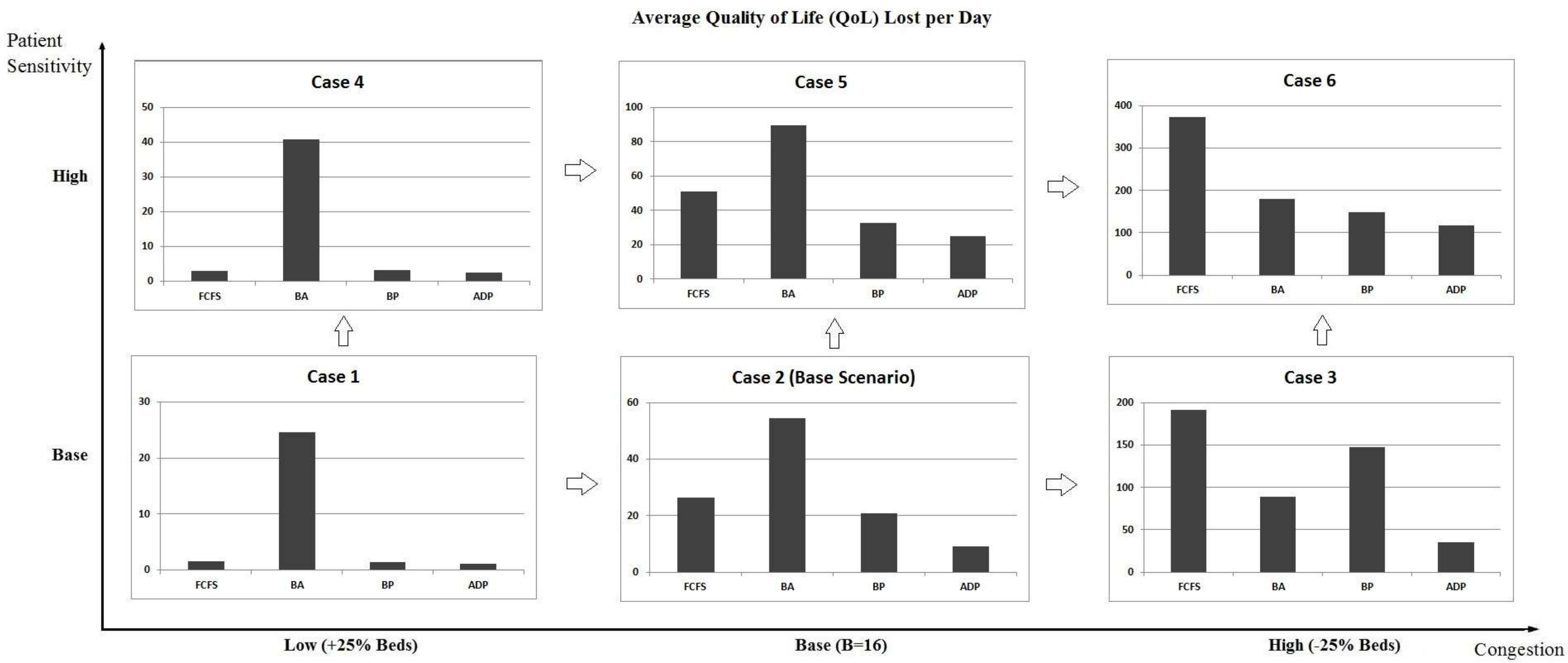
- Four types of patient in MNH.

Parameters (per day)	Mild non-Stroke	Mild Stroke	Severe non-Stroke	Severe Stroke
Arrival Rate	0.2362	0.2625	0.1388	0.1125
Discharge Rate	0.07690	0.0870	0.0526	0.04545
Waiting Cost	70	90	145	295

- Base scenario (B=16, $\boldsymbol{\kappa} = 2\boldsymbol{\pi}$).
- Alter the service capacity by $\pm 25\%$ (3 cases including the base scenario).
- Increase the cost: $\boldsymbol{\pi} = (70,90,500,600)$ and $\boldsymbol{\kappa} = 3\boldsymbol{\pi}$ (3 cases).

Policy Alternatives Performance

- Compare the ADP policy to BA, BP and FCFS policies



Policy Alternatives Performance

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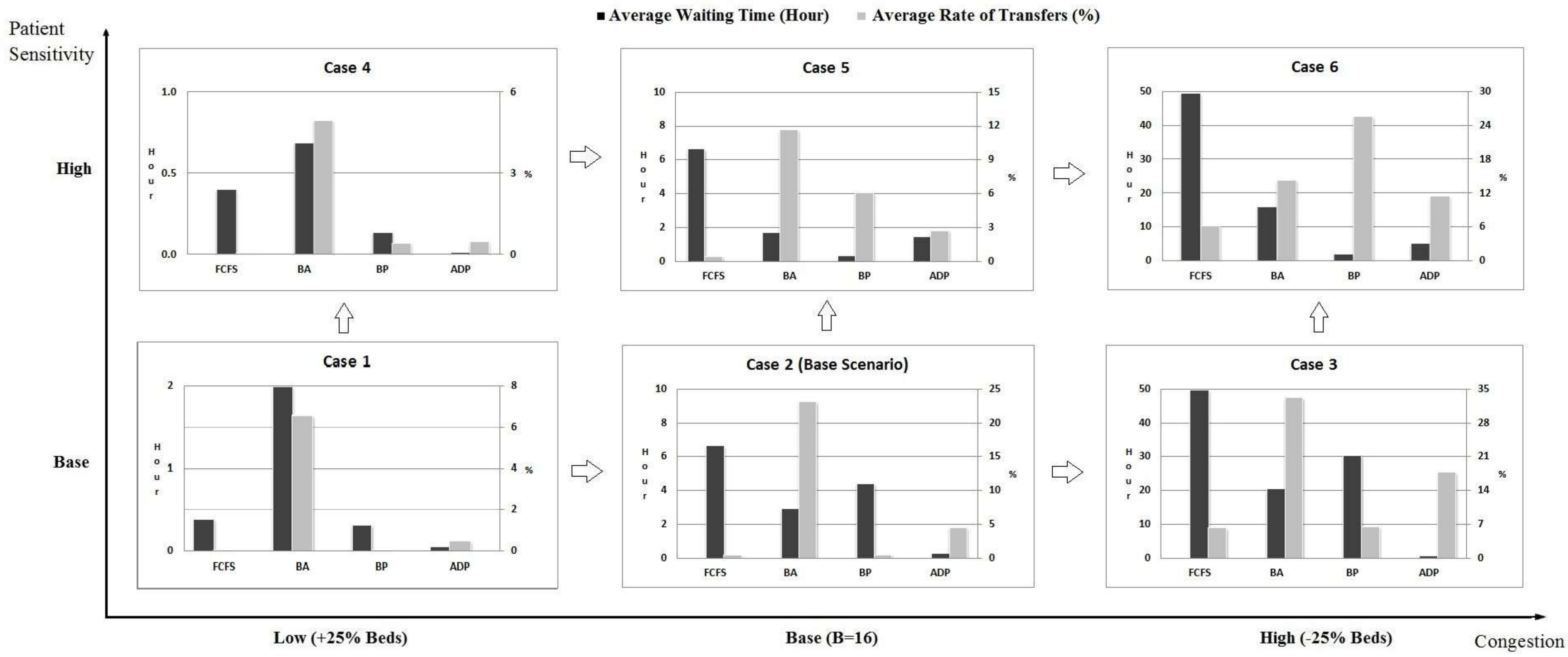


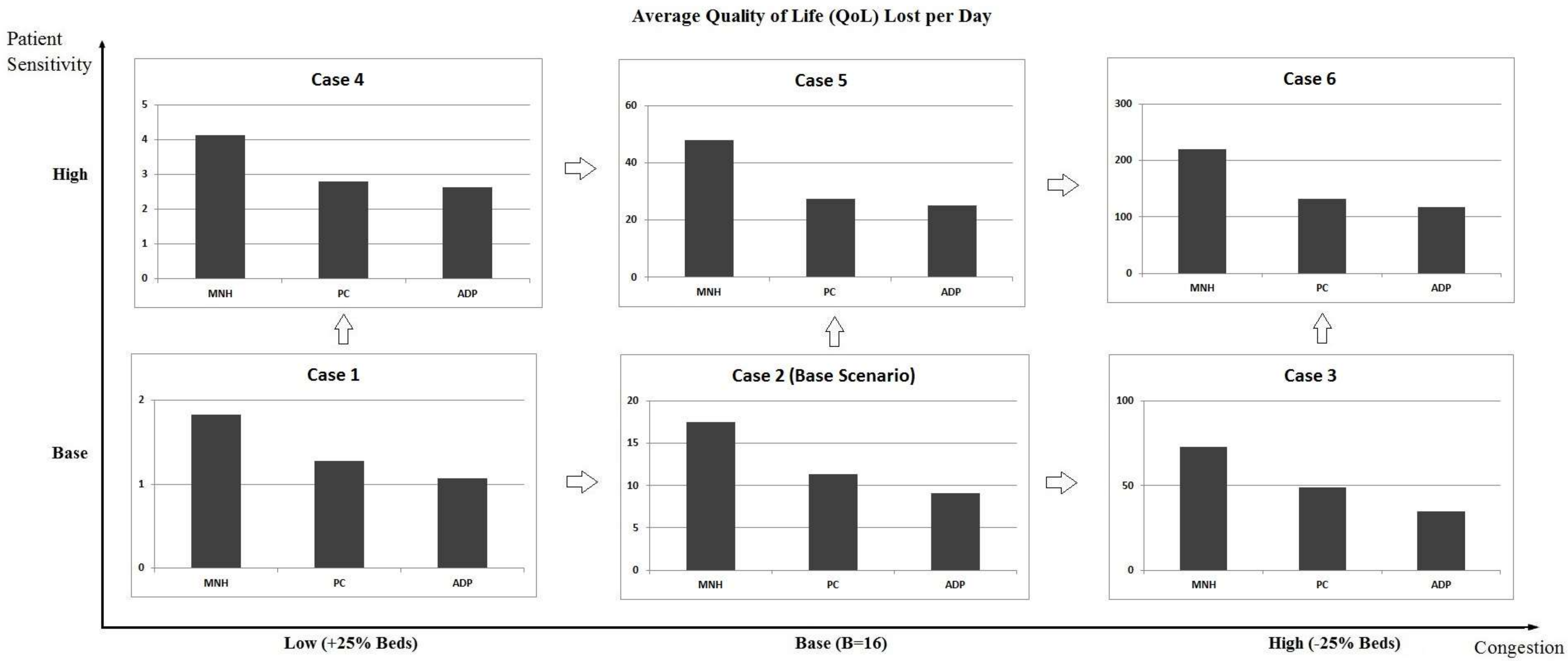
A More Practical Policy

An ADP-based Heuristic Policy

- The ADP policy is challenging to implement.
- Focus on the system states that are critical.
- Develop a policy that follows the ADP policy in critical states and a simple policy (such as FCFS) in other states.
- Group patients based on the severity regardless of disease.
- Observations from ADP policy in critical states:
  a) The ADP policy tends to reserve *some* beds for severe patients.
  b) The ADP policy seems to evaluate the chance of a discharge in the near future while making decision about admitting or transferring.
  c) The chance of a discharge depends on the patient mix in the ward.
- ADP-based **P**riority **C**ut-off (PC) policy.

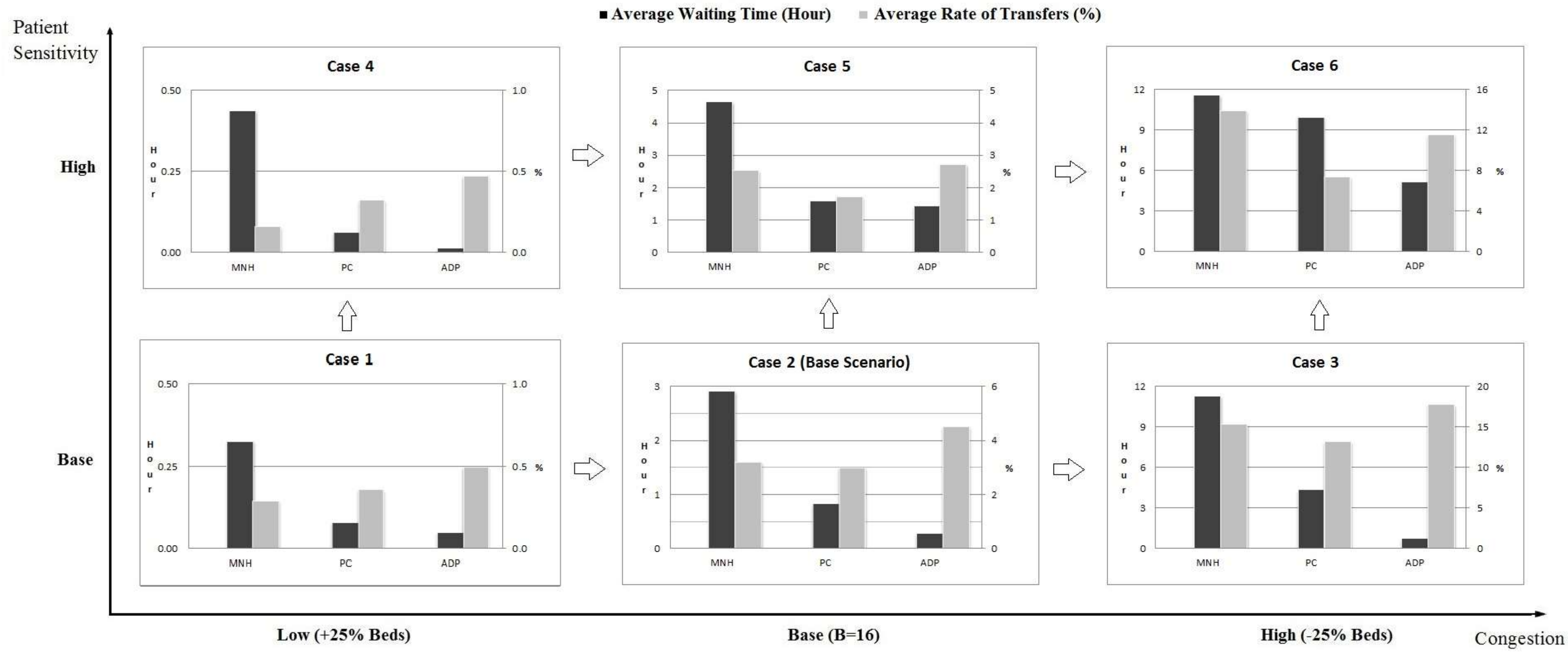
Policy Alternatives Performance

- Current policy in MNH: (6-6-4) and 48*hr* Transfer
- Comparison of ADP, PC and MNH policies



Policy Alternatives Performance

- Comparison of ADP, PC and MNH policies



Conclusion

Summary

- This is the first paper that makes an explicit effort to model the specific features of neurology wards.
  - Problem comes from practice.
  - Real data available to estimate model parameters and evaluate the policy implications.
- We propose an easy-to-implement heuristic policy based on the ADP that performs well compared to other heuristic policies.
- Insights for hospital mangers:
  - Make pro-active transfer decision.
  - Dedicating beds to patient types can lead to poor performance of the neurology ward.
  - Use an earmarking strategy based on the level of severity of the patients instead of their diseases (like PC policy).

Summary

- We develop an approximate dynamic programming approach based on nonlinear functional approximations for an infinite-horizon continuous-time average-cost problem.
  - An LP-based ADP that does not require solving a large LP.
  - Potential applications for general multi-class queueing control problems.
- Limitations:
  - Readmission to ICU after being admitted to ward.
  - The impact of waiting on LOS.
  - Patients who are not admitted through ED.

Thanks for your attention!

Questions\Comments?

Appendices

Related Literature

Area	Papers
Patient Admission & Bed Allocation Problem	Kolesar (1970) Esogbue and Singh (1976) Ksuters and Groot (1996) Lapierre et al (1999) Gorunesco et al (2002.a, 2002.b) Akcali (2006) Li et al (2008) Bekker and de Bruin (2010) Ayvaz and Huh (2010) Helm et al (2011) Mandelbaum (2012)
ADP in Healthcare	Green et al (2006) Patrick et al (2008) Saure et al (2012)

Differentiating Characteristics

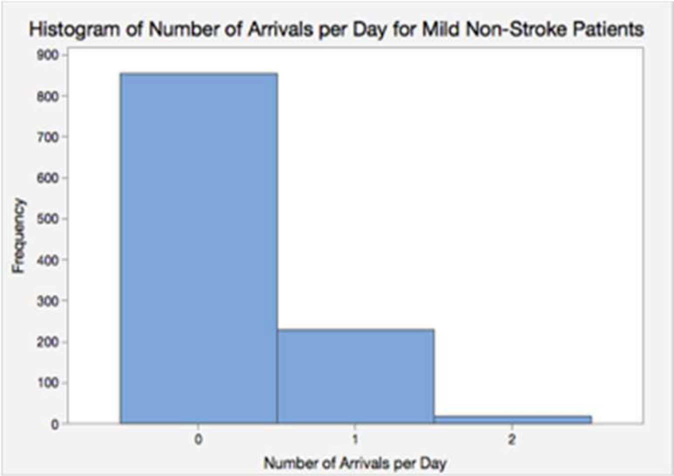
1. We recognize the significance of the presence of a specialized team of care;
   a. Which renders off-unit servicing infeasible.
   b. All types of patients can wait for service as long as there is space available in the waiting area (i.e. ED).
   c. We incorporate the decision about transferring the patients to another hospital.
2. We consider the different length of stay associated with each patient type.
3. We combine queueing methods and approximate dynamic programming (ADP) in devising an integrated solution procedure:
   a. LP-based approximate dynamic programming but does not require solving large-scale LP.
   b. Non-linear functional approximation.

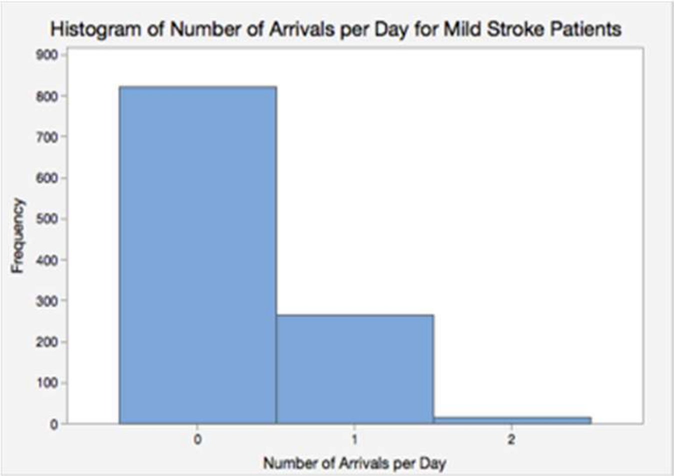
Arrival and LOS

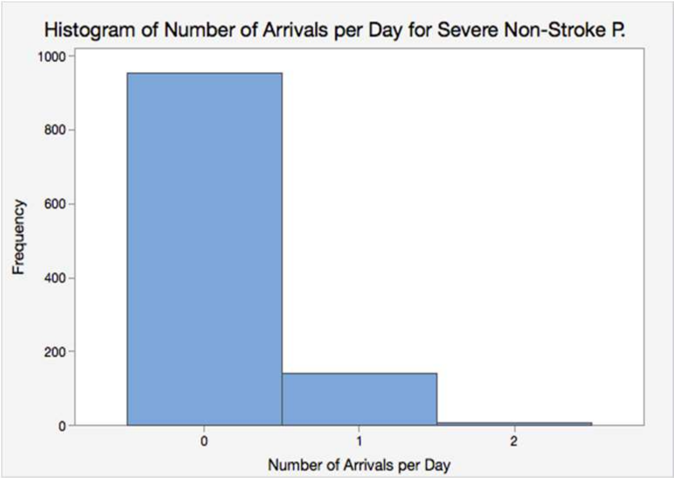
- Three full years of data from MNH (Patient registry system, patients' chart review, ED information system).
- Arrivals: χ^2 goodness-of-fit test.
- LOS: Anderson-Darling goodness-of-fit test.

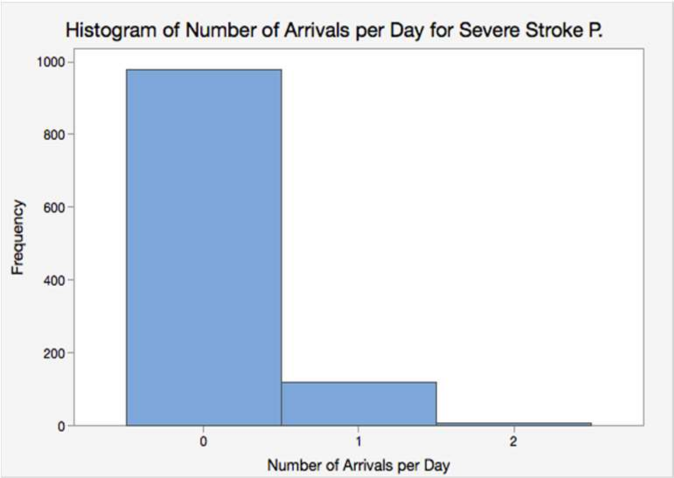
Patient Type	Arrival Process (per day)			Length of Stay (day)		
	Mean (λ)	Variance	H_0 : Arrival is Poisson	Mean ($^1/_\mu$)	Variance	H_0 : LOS is Exponential
Mild Non-Stroke	0.236	0.246	Not Rejected ($0.05<p<0.10$)	13.003	162.383	Not Rejected ($0.05<p<0.10$)
Mild Stroke	0.262	0.291	Not Rejected ($0.10<p<0.25$)	11.491	114.204	Not Rejected ($0.05<p<0.10$)
Severe Non-Stroke	0.139	0.141	Not Rejected ($0.25<p<0.50$)	19.011	305.904	Not Rejected ($0.25<p<0.50$)
Severe Stroke	0.113	0.117	Not Rejected ($0.25<p<0.50$)	22.002	596.445	Not Rejected ($0.50<p$)

Arrivals (Histograms)









Arrivals (Distributional Assumptions)

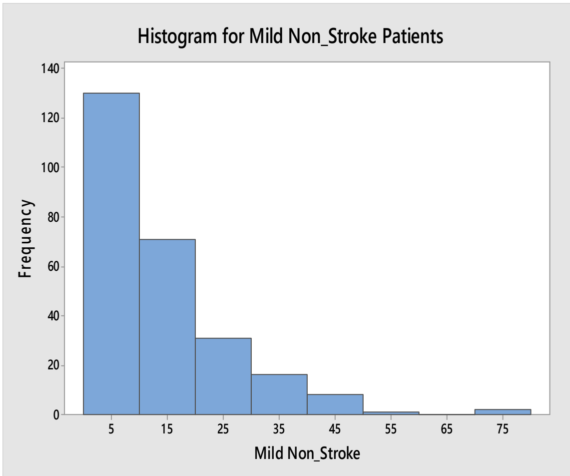
- Three full years of data (Patient registry system, patients' chart review, and ED information system).
- χ^2 goodness-of-fit test with bin size=1

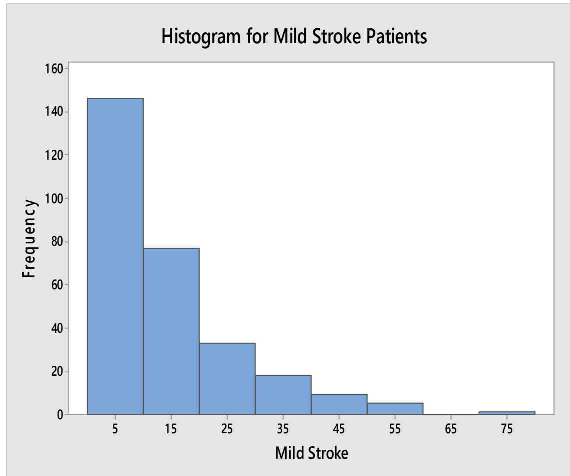
Patient Type	Sample Size	Arrival Process (per day)		
		Mean (λ)	Variance	H_0 : Arrival is Poisson
Mild Non-Stroke	259	0.236	0.246	Not Rejected (*p-value:* **0.097**)
Mild Stroke	289	0.262	0.291	Not Rejected (*p-value*: **0.165)**
Severe Non-Stroke	151	0.139	0.141	Not Rejected (*p-value:* **0.395**)
Severe Stroke	123	0.113	0.117	Not Rejected (*p-value*: **0.401)**

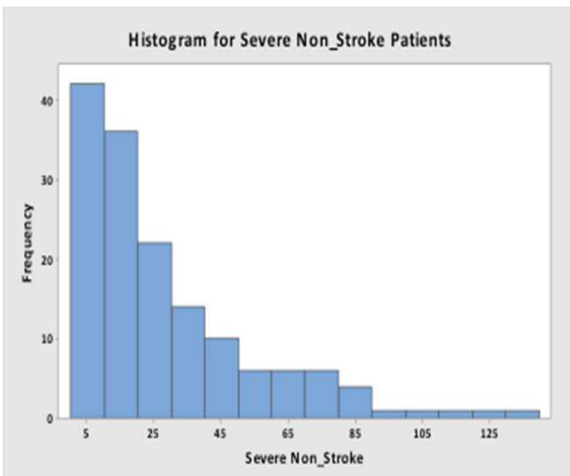
Arrivals (Stationarity Assumptions)

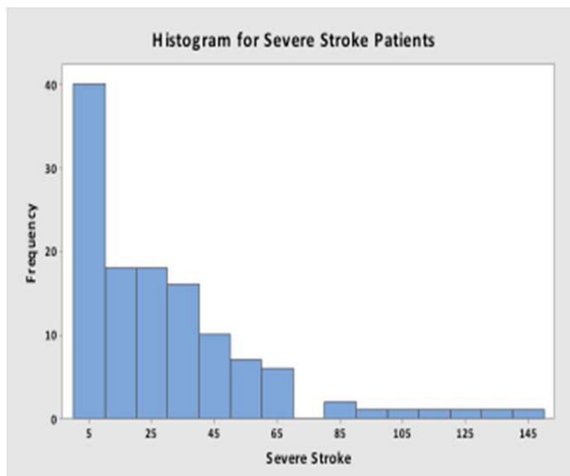
- Examined if rates vary with time of the day, day of the week, or month of the year using Poisson regression analysis.
- 6-hour intervals for each patient type.
- All the *p*-values corresponding to all variables (time of the day, day of the week, or month of the year) are greater than 0.05.
- Extended analysis: variables for weekday\weekend and season.
- The results of this analysis also confirm that the rates of arrivals do not vary either with the weekday\weekend or the season.

Length of Stay (Histograms)









Length of Stay (Distributional Assumptions:

- Anderson-Darling goodness-of-fit test.

Patient Type	Sample Size	Length of Stay (day)		Exponential		Weibull		Gamma		Lognormal	
		Mean ($^1/_\mu$)	Variance	AD	p-value	AD	p-value	AD	p-value	AD	p-value
Mild Non-Stroke	259	13.0	162.5	1.13	0.084	1.06	<0.01	1.16	0.007	2.30	<0.005
Mild Stroke	289	11.5	114.2	1.23	0.064	1.33	<0.01	1.45	<0.005	2.44	<0.005
Severe Non-Stroke	151	19.0	305.9	0.72	0.263	0.74	0.05	0.79	0.048	0.81	0.035
Severe Stroke	123	22.0	596.5	0.43	0.589	0.42	>0.250	0.43	>0.250	1.76	<0.005

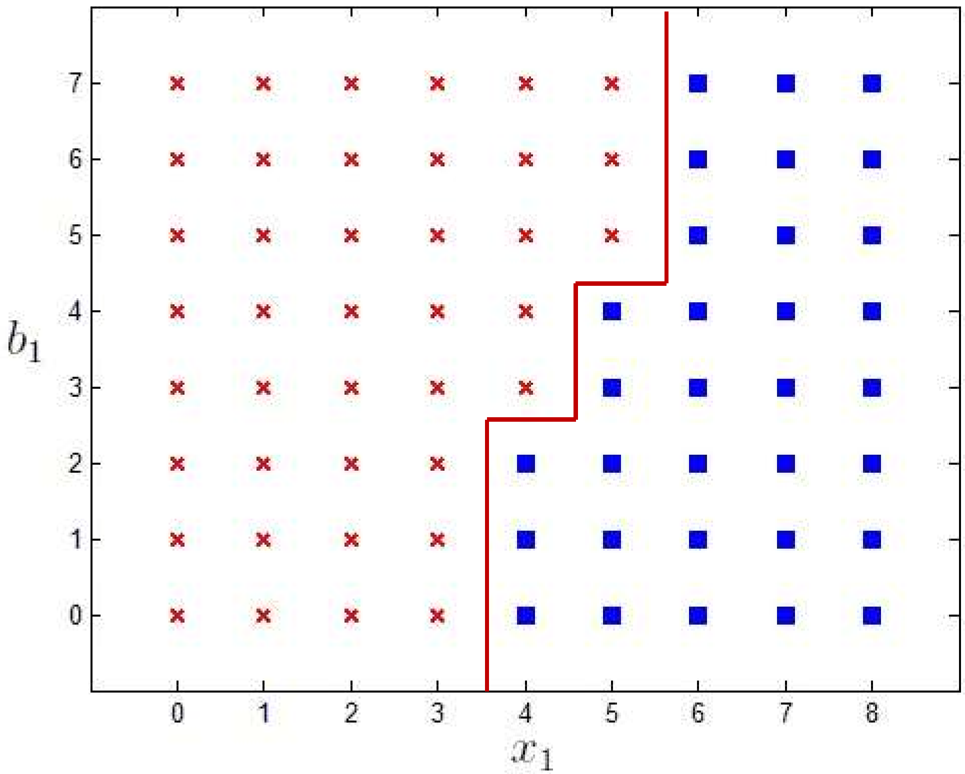
Illustrative Examples

Illustrative Example I

- Mild and severe stroke patients ($\pi_2 = 5\pi_1$)
- Arrival of a mild stroke patient when only one bed is available ($b_1 + b_2 = B - 1$) and $x_2 = 0$:

x : Transfer

■ : Admit a mild patient to the ward

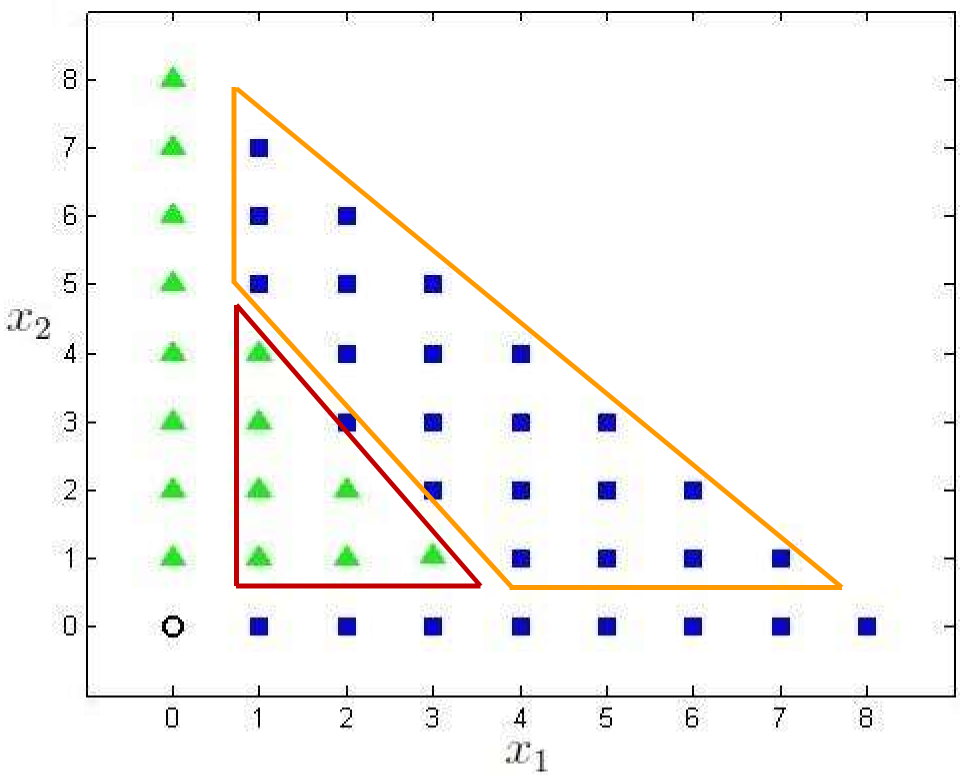


Illustrative Example II

- Mild and severe stroke patient ($\pi_2 = 1.25\pi_1$)
- Discharge of a mild patient when all beds are occupied $\left(b_1 = b_2 = \frac{B}{2}\right)$:

■ : Admit a mild patient to the bed

▲ : Admit a severe patient to the bed



The Bed Allocation Policy

- Inspired by **SM.**
- Static admission policy.

The **B**ed **A**llocation (**BA**) Policy:

1. Admit a new arriving type-i patient if the number of occupied beds by these patients is less than $\tilde{b}_i^*$.
2. When all $\tilde{b}_i^*$ beds are occupied, and there is room available in the ED, admit the new arrival to the queue with probability of $p_i = \frac{\tilde{\lambda}_i^*}{\lambda_i}$ and transfer with probability of $1 - p_i$.

The Bid Price Policy

- Inspired by **SM** and Revenue Management literature.
- Semi-dynamic admission policy.

The **B**id **P**rice (**BP**) Policy:

1. If there is at least one bed available, admit an arriving type-i patient to the ward if $\alpha\mu_i^{-1} \leq \kappa_i$ and transfer otherwise.
2. If there is no bed available, admit a new arriving patient of type-i to the queue if $\alpha\mu_i^{-1} + \pi_i W_i^* \leq \kappa_i$ and transfer otherwise.
3. If one bed becomes available, priority is given to the patients with highest waiting cost.

The Priority Cut-off Policy

- Inspired by the ADP policy.
- Dynamic admission policy.

The ADP-based **P**riority **C**ut-off (**PC**) Policy:

1. When a severe patient arrives:
    a) If at least one bed is available, admit the patient to the ward.
    b) If all beds are occupied,
        i. transfer the patient in the case of a small transfer cost.
        ii. admit the patient to the queue if the chance of a discharge is high and transfer the patient otherwise in the case of a large transfer.

The Priority Cut-off Policy

The ADP-based **P**riority **C**ut-off (**PC**) Policy: (cont'd)

2. When a mild patient arrives:
   a) If more than S beds are available, admit the patient to the ward (i.e. FCFS).
   b) If between one and S beds are available:
      i. admit the patient to the ward if the chance of a discharge is high,
      ii. admit the patient to the queue if the chance of a discharge is medium,
      iii. transfer the patient if the chance of a discharge is low.
   c) If all beds are occupied, admit the patient to the queue if the chance of a discharge is high and transfer the patient otherwise.
3. If a discharge occurs, the priority of admitting a patient to the ward is always given to the severe patients. If no severe patient is waiting in the queue, admission of a mild patient follows item 2.

Non-stationary Arrival

- Our methodology can be adapted to problem settings with non-stationary arrival processes by building on point-wise stationary approximation (PSA).
- Examine the performance of PSA by considering a weekly cyclic pattern for arrivals.
- Robustness of the ADP policy with respect to non-stationary arrivals (average QoL lost per day in the table):

Case	Policy					ADP
	FCFS	BA	BP	PC	ADP	Improvement
1	2.84	36.36	2.42	1.81	1.59	13%
2	29.37	60.24	24.22	12.06	9.99	17%
3	170.70	89.50	83.61	64.83	63.26	2%
4	5.44	341.20	4.60	5.09	3.92	15%
5	56.59	116.69	54.93	57.12	55.74	-1%
6	333.61	151.96	140.85	146.33	140.99	-0.1%

Non-linear Waiting Cost

- It seems more realistic to assume the patient's health status deteriorates at higher rates when the waiting time increases.
- Consider piecewise-linear increasing convex functions for the waiting costs of patients (with 3-hour time intervals with increasing slope)
- Use the slope of a linear function fitted to the piecewise-linear function and our solution approach.
- In the table: the percent increase in the total cost associated with each policy, when the waiting costs are incurred according to a non-linear function:

Case	Cost Increase (%)					ADP Improvement (%)	
	FCFS	BA	BP	PC	ADP	Linear	Non-linear
1	63	73	63	41	29	27	42
2	98	53	94	56	33	57	70
3	134	77	126	78	35	61	71
4	64	102	76	96	91	12	-3
5	99	102	92	114	73	24	31
6	137	131	98	151	87	22	26